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ON THE INFLUENCE OF INHOMOGENEITIES ON THE VIBRATIONS OF A CYLINDRICAL SHELL*

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The dynamic characteristics of a cylindrical shell (CS) of unlimited length with an inhomogeneity stiffly clamped along the shell generator distributed along the azimuth are analysed. The solution obtained enables one to represent the expression for the eigenmodes and frequencies of the low-frequency azimuthal branch of the vibrations of an inhomogeneous CS in the approximations of technical theory in analytic form. The influence of the inhomogeneity on the distribution of the vibrational velocity is clarified, which enables the vibration-acoustical characteristics of inhomogeneous systems of this kind to be diagnosed.

A number of papers /1-3/ have been devoted to the analysis of the vibrational characteristics of thin CS with inhomogeneities of the attached-mass type. However, the problems of the vibrational characteristics associated with changes in the vibrational velocity distribution under the action of an inhomogeneity have been studied in much less detail.

1. The vibrations of an infinite cylindrical shell (CS) along whose generator an inhomogeneity characterized by a mass per unit length m_0 and a moment of inertia I_0 with respect to rotation through an angle ϑ around the normal to the shell surface (for $\varphi = \pi$) are examined. The azimuthal dimensions of the inhomogeneity are represented by the parameter $\eta = \varphi_0/\pi$ $(0 \leqslant 2\varphi_0 \leqslant \pi/2)$, and the centre of inertia of the inhomogeneity is found for $\varphi = \pi$.

We will limit ourselves to the simplest case corresponding primarily to the low-frequency azimuthal modes of CS vibrations, and we will neglect the tangential forces of shell inertia. We will investigate the vibrational characteristics ignoring the reaction of the intrinsic radiation field (which occur for CS vibrations in air). Then by using the approximation of technical shell theory we obtain the following system of equations of the selfconsistent problem of forced vibrations of an inhomogeneous CS in an elastic medium by using the Ostrogradskii-Hamilton principle

$$
\begin{gathered}
W''' - \xi^2 W = F/\beta^2 \\
(W''|_{\varphi=\pi-\varphi_0} - W''|_{\varphi=-\pi+\varphi_0}) + {}^1/_2 \sin 2\pi\eta\, (W'''|_{\varphi=\pi-\varphi_0} + \\
W'''|_{\varphi=\pi+\varphi_0}) + 2\pi\alpha H^2\xi^2 W' = M^{(N)}/(A\beta_0^2) \\
(W'''|_{\varphi=\pi-\varphi_0} - W'''|_{\varphi=-\pi+\varphi_0}) + \pi\alpha\xi^2 (\cos\pi\eta)^{-1} (W|_{\varphi=\pi-\varphi_0} + W|_{\varphi=-\pi+\varphi_0}) = \\
F^{(N)}a/(A\beta^2)
\end{gathered} \tag{1.1}
$$

where W is the radial CS displacement and the prime denotes a derivative with respect to φ.

The system (1.1) is solved simultaneously with the equations corresponding to rigid clamping of the inhomogeneity on the shell surface

$$
\begin{gathered}
r = a,\ W|_{\varphi=\pi-\varphi_0} - W|_{\varphi=-\pi+\varphi_0} = -\vartheta a \sin 2\pi\eta, \\
W'|_{\varphi=\pi-\varphi_0} = W'|_{\varphi=-\pi+\varphi_0} = a\vartheta
\end{gathered} \tag{1.2}
$$

where $F = F(\varphi)$ is the intensity of the external harmonic force; in the case of a local force applied at the point $\varphi = \varphi_1$, we have $F = F^{(l)}a^{-1}\delta(\varphi - \varphi_1)$, and $F^{(N)}$, $M^{(N)}$ are the external force and the moment of the force applied to the inhomogeneity. The following dimensionless parameters are used in systems (1.1) and (1.2)

$$
\begin{gathered}
\zeta = \omega/\omega,\ \alpha = m_0/(2\pi m_s a),\ \varkappa = I_0/(2\pi m_s a^3) = \alpha H^2 \\
I_0 = m_0 h_s^2,\quad H = h_s/a,\ A = Eh/(1 - \nu^2) \\
(\omega_1 = \omega_0\beta,\ \beta = h/(\sqrt{12}a),\ \omega_0 = a^{-1}\sqrt{E/(\rho_s(1 - \nu^2))},\quad m_s = \rho_s h)
\end{gathered} \tag{1.3}
$$

Here ξ is the dimensionless frequency of the external harmonic force, ω_0 is the frequency of the pulsating shell vibrations, α is a parameter of the inhomogeneity equal to the ratio between the mass per unit length of the inhomogeneity and the shell of radius a, m_s is

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the mass per unit of surface of a shell of thickness h, $\varkappa$ is a parameter characterizing the moment of inertia of the inhomogeneity, h_s is the effective dimension of the inhomogeneity, and E, ν, ρ_s are Young's modulus, Poisson's ratio,,and the density of the shell material, respectively.

The system of equations can be solved by well-known methods by representing the solution in the form of series in the eigenmodes of the vibrations corresponding to a homogeneous CS. This results in a system of algebraic equations in the amplitudes of the vibrations that characterize a system of coupled harmonic oscillators between which the coupling occurs both via the surrounding elastic medium as well as via the inhomogeneity. The magnitude of the coupling increases as the inhomogeneity parameters α and $\varkappa$ incrase; consequently, an analysis of the forced vibrations and radiation of an inhomogeneous CS is possible only by numerical methods.

In order to clarify the influence of the inhomogeneity on the nature of the forced vibrations it is best to represent the solution in the form of a series expansion in the eigenmodes of the vibrations of an inhomogeneous system. In this case, a complete orthonormal system of eigenfunctions can be constructed whose fundamental characteristics are obtined successfully in analytic form, which enables us to give a graphic physical interpretation of the influence of the inhomogeneity on the vibrational-acoustic characteristics of a CS.

Taking into account that the inhomogeneous system has a plane of symmetry at $\varphi = 0$ the solution of problem (1.1) and (1.2) can be represented in the form

$$W(\varphi) = W^{(1)}(\varphi) + W^{(2)}(\varphi) \tag{1.4}$$

where $W^{(1)}(\varphi)$ is a symmetric function and $W^{(2)}(\varphi)$ an antisymmetric function, which permits separation of the system of Eqs.(1.1) and (1.2) into symmetric and antisymmetric parts.

The solution of these equations has the form

$$W_q^{(1)} = \cos\Phi - \frac{\sin\zeta}{\operatorname{sh}\zeta}\operatorname{ch}\Phi, \quad W_q^{(2)} = \sin\Phi - \frac{\sin\zeta + \chi\cos\zeta}{\operatorname{sh}\zeta + \chi\operatorname{ch}\zeta}\operatorname{sh}\Phi \tag{1.5}$$
$$-\pi + \varphi_0 \leqslant \varphi \leqslant \pi - \varphi_0$$
$$\chi = (\gamma/\pi)\sin 2\pi\eta, \quad \zeta = \gamma(1-\eta), \quad \Phi = \gamma\varphi/\pi$$

where $\gamma = \gamma_q^{(i)}$ are the eigenvalues and $\xi_q^{(i)}$ are the eigenfrequencies; $\xi_q^{(i)}$ are given for symmetric $(i = 1)$ eigenfunctions by the relations

$$1 + (\alpha \cdot {}^1/_2\gamma/\cos\pi\eta)[\operatorname{ctg}\zeta, \ \operatorname{ctg}\zeta + \operatorname{cth}\zeta] = 0, \quad \xi_q^{(1)} = (\gamma_q^{(1)}/\pi)^2 \tag{1.6}$$

and for the antisymmetric $(i - 2)$ eigenfunctions by

$$1 - (\gamma/\pi)^2\sin^2 2\pi\eta\cdot\operatorname{ctg}\zeta, \operatorname{ctg}\zeta + \varkappa(\gamma^3/(2\pi^2))(\operatorname{ctg}\zeta - \operatorname{cth}\zeta) = 0, \tag{1.7}$$
$$\xi_q^{(2)} = (\gamma_q^{(2)}/\pi)^2$$

Here $q = 2, 3$ is the number of roots of the appropriate characteristic equations.

The radial displacement in the range $\pi - \varphi_0 \leqslant \varphi \leqslant -\pi + \varphi_0$ can be determined from the geometrical relations

$$W_q^{(1)} = (W_q|_{\varphi=\pi-\varphi_0}\cos\varphi)/\cos\pi\eta, \quad W_q^{(2)} = (W_q|_{\varphi=\pi-\varphi_0}\sin 2\varphi)/\sin 2\pi\eta \tag{1.8}$$

Expressions (1.5) and (1.8) determine the eigenfunctions $W_q^{(i)}$ of the CS in the range $-\pi \leqslant \varphi \leqslant \pi$. The orthogonality of $W_q^{(i)}$, whose weight characteristic depends on the inhomogeneity parameters $\alpha, \varkappa, \eta$ can be proved by a well-known method.

The norm $D_q^{(i)}$ of the symmetric and antisymmetric vibrations modes is calculated.

We shall later use the orthonormal eigenfunctions

$$\psi_q^{(i)}(\varphi) = W_q^{(i)}(\varphi)/\sqrt{D_q^{(i)}}, \quad i = 1, 2 \tag{1.9}$$

2. Let us examine the characteristic singularities of the eigenfunctions $\psi_q^{(i)}$ and eigenvalues $\gamma_q^{(i)}$ $(i = 1, 2)$. For small values of the inhomogeneity parameters $\alpha \ll 1, \varkappa \ll 1$ (1.3) the solutions of the characteristic Eqs.(1.7) and (1.8) have the form

$$\gamma_q^{(1)} \simeq \frac{q\pi}{1-\eta}\left[1 - \frac{\alpha}{2(1-\eta)\cos\pi\eta}\right] \tag{2.1}$$
$$\gamma_q^{(2)} \simeq \frac{q\pi}{1-\eta}\left[1 + \frac{q\sin^2 2\pi\eta - 4\varkappa q^2\pi}{8\pi}\right]; \quad q = 2, 3, 4$$

Note that in the limit case when $\alpha \to 0$ and $\eta \to 0$ (the inhomogeneity shrinks to a point) $\gamma \to \pi q$, $\sin\zeta = 0$, $\chi = 0$ the eigenvalues and eigenfunctions are

$$\gamma_q^{(i)} = q\pi, \quad \xi_q^{(i)} = q^2 \ (i = 1, 2); \quad \psi_q^{(1)} = \pi^{-1/2}\cos q\varphi, \tag{2.2}$$
$$\psi_q^{(2)} = \pi^{-1/2}\sin q\varphi$$

which is identical with the well-known relationships for a homogeneous CS.

Therefore, the presence of an inhomogeneity results in a reduction in the degeneration of the eigenvalues for a CS when identical eigenvalues γ_q and eigenfrequencies ξ_q (2.2) correspond to eigenfunctions of the type $\cos q\varphi$ and $\sin q\varphi$.

It follows from relationships (1.5)-(1.7), (2.1) that the mass of the inhomogeneity (the parameter α) exerts its influence primarily on the symmetric modes of vibrations and the moment of inertia of the inhomogeneity (the parameter $\varkappa$) on the antisymmetric modes; the parameter characterizing the azimuthal dimension of the inhomogeneity (η) exerts its influence on both the symmetric and antisymmetric vibrations modes. For $\alpha \neq 0, \varkappa \neq 0$ a reduction in the eigenfrequency values occurs for the system $\xi_q^{(1)} \neq \xi_q^{(2)}$.

The solution of the characteristic Eqs.(1.6) and (1.7) is obtained successfully in the important practical case of large values of the inhomogeneity parameters $\alpha > 1, \varkappa > 1$, when

$$\gamma_q^{(1)} = \frac{Q_1/\pi}{1-\eta}\left[1 + \frac{(1-\eta)\cos\pi\eta}{\alpha\pi^2 Q_1^2}\right]$$
$$\gamma_q^{(2)} = \frac{Q_2/\pi}{1-\eta}\left[1 + \frac{Q_2^2\sin^2 2\pi\eta - 4}{2\pi\left(Q_2^3\sin^2 2\pi\eta - 2\pi\varkappa Q_2^4\right)}\right] \tag{2.3}$$

As computations show, relations (2.3) determine the eigenvalues with less than 5% error for $\alpha, \varkappa \geqslant 2$, which enables these relations to be used for a clear interpretation of the influence of inhomogeneities on the CS vibrational characteristics.

It follows from relations (2.1) and (2.3) that an inhomogeneity changes the eigenvalues of the q-th mode of vibrations within the limits

$$q/(1-\eta) < \gamma_q^{(i)}/\pi < Q_i/(1-\eta), \quad i = 1, 2 \tag{2.4}$$

i.e., the moment of inertia (the parameter $\varkappa$) of the inhomogeneity exerts a stronger influence on the change $\gamma_q^{(2)}$ compared with the influence of the mass of the inhomogeneity (the parameter α) on $\gamma_q^{(1)}$.

An increase in the azimuthal dimensions of the inhomogeneity (η) results in an increase in the eigenvalues for both the symmetric and antisymmetric modes of vibration, where even for large values of α the reduction in the frequency can be compensated for the symmetric modes because of the increase in η. For instance, for $\alpha = 1$, $\eta = 0.01$ the value γ_q for the second mode is identical with the eigenvalues of a homogeneous system.

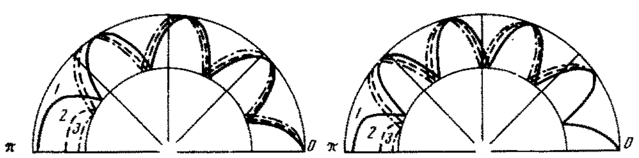

Fig.1

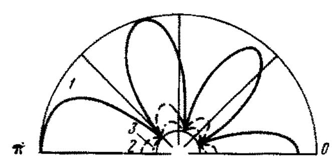

Fig.2

Relations (2.1)-(2.4) govern the characteristic deformation singularities of the eigenmodes of CS vibrations, as can be seen in Fig.1, where the moduli of four symmetric modes of vibration are presented for $\eta = 0.1$ (on the left) and five symmetric modes for $\eta = 0.05$ (on the right). Curves *1*, *2* and *3* correspond to the values $\alpha = 0.02, 0.2, 1$.

A decrease in the distances between the nodes $\psi_q^{(1)}$ occurs in the neighbourhood of the inhomogeneity due to the effect of the inhomogeneity, and a reduction in the antinode amplitude occurs at the point where the inhomogeneity is situated $\psi_q^{(1)}(-\pi + \varphi_0; \pi - \varphi_0)$ as compared with the corresponding parameters for a homogeneous CS (2.2).

Values of the coefficients of the expansions of $\psi_q^{(i)}$ in Fourier series in the eigenfunctions of a homogeneous system (symmetric and antisymmetric)

$$\psi_q^{(1)}(\varphi) = \frac{aq_0}{2} + \sum_{m=1}^{\infty} a_{qm}\cos m\varphi, \quad \psi_q^{(2)}(\varphi) = \sum_{m=1}^{\infty} b_{qm}\sin m\varphi \tag{2.5}$$

can be more convenient measure of the degree of eigenfunction deformation under the action of an inhomogeneity for practical purposes.

The influence of the inhomogeneity parameters on the deformation of the eigenfunctions $\psi_q^{(1)}$ can be traced from the dependence of a_{qm} on α and η. As α increases, the values of a_{qm} increase and tend to constant values for fairly large α. An increase in the parameter η results in a reduction in the values of the expansion coefficients a_{qm}. It should also be noted that the coefficient $a_{20}=0$, for certain combinations of the parameters α and η (for example, for $\eta=0.1$, $\alpha=0.15$). It is characteristic that the inhomogeneity results in such a deformation of the eigenfunctions when terms corresponding to pulsating $(m=0)$ and oscillating $(m=1)$ CS vibrations occur in expansions of the symmetric eigenfunctions $\psi_q^{(1)}(\varphi)$ with numbers $q\geqslant 2$. Terms of the expansion corresponding to oscillating $(m=1)$ homogeneous shell vibrations occur for antisymmetric eigenfunctions $\psi_q^{(2)}(\varphi)$ for $q\geqslant 2$. This circumstance is of practical importance for problems concerning inhomogeneous CS radiation.

3. We will analyse the forced vibrations of an inhomogeneous CS by a known method by representing the solution of the problem of forced vibrations in the form of a series expansion in the eigenmodes of the system vibrations in a vacuum

$$v(\varphi)=v_0\sum_{q=2}^{\infty}[c_q^{(1)}\psi_q^{(1)}(\varphi)+c_q^{(2)}\psi_q^{(2)}(\varphi)] \tag{3.1}$$

where v_0 is the dimensional amplitude and $c_q^{(i)}$ are the desired amplitudes of the vibrational velocity expansion.

Substituting the expansion (3.1) into (1.1) and using the orthogonality property of the eigenfunctions $\psi_q^{(i)}$, the solution of this system of equations in the amplitude $c_q^{(i)}$ can be obtained in the form

$$c_q^{(i)}=\frac{\psi_q^{(i)}(\varphi)}{Z_q^{(i)}},\quad Z_q^{(i)}=\frac{\xi_q^{(i)2}-\xi^2}{-i\xi},\quad v_0=\frac{F_0}{(m_s\omega_1 a)} \tag{3.2}$$

The solution (3.2) is obtained under the assumption that a local force $F=F_0a^{-1}\delta(\varphi-\varphi_1)$, acts on the shell, where φ_1 is the point of application of the force. We note that the solutions represented in the form of a series expansion in eigenfunctions of the inhomogeneous CS result in a diagonal system of equations in $c_q^{(i)}$ and we obtain the solution (3.2) when there is no interaction between the CS modes of vibration. Such interaction occurs only when the reaction of the medium to the CS vibrations is taken into account.

If the solution of the problem of forced CS vibrations is represented in the form of series expansions in the eigenmodes of homogeneous CS vibrations

$$v=v_0\sum_{m=0}^{\infty}(A_m\cos m\varphi+B_m\sin m\varphi) \tag{3.3}$$

then the system of equations in the amplitudes A_m and B_m will differ from the diagonal system, which is associated with interactions between the modes of the homogeneous CS vibrations in terms of the existing inhomogeneity. The solution of the system the equations in the amplitudes A_m and B_m can be obtained only by numerical methods, which makes a physical interpretation of the influence of the inhomogeneity on the CS vibrational characteristics difficult.

The solution (3.1), (3.2) enables us to clarify the fundamental singularities of this influence. Indeed, relationships (1.6), (1.7), (3.1) and (3.2) determine the dependence of the vibrational velocity on the frequency and azimuth $v(\xi,\varphi)$. The frequency dependence of the vibrational velocity has a sequential series of minima when the resonance conditions $\xi=\xi_q^{(i)}$ (3.2) are satisfied.

Relationships (3.1) and (3.2) allow a graphical illustration of the influence of the point of application of the force on the vibrational characteristics of the system. For instance, in the case of practical importance when the local force is applied at the centre of the inhomogeneity $(\varphi_1=\pi)$, only symmetric modes of vibration will be excited for which $\psi_q^{(i)}(\varphi=\pi)\neq 0$, $(i=1,2)$. If the point of application of the local force does not coincide with the centre of the inhomogeneity $(\varphi_1\neq\pi)$, a moment of the force appears and both symmetric and antisymmetric modes of vibration will be excited provided that φ_1 does not coincide with the location of the angle in the vibratational velocity distribution. Therefore, the selection of the method of exciting the system can exert a substantial influence on the value of the amplitudes $c_q^{(i)}$ and, therefore, on the vibrational acoustic characteristics of the inhomogeneous CS.

Graphs of $v(\varphi)$ as a function of the azimuthal angle at the resonance frequencies $\xi=\xi_q$ for $\xi_q=3$ are shown in Fig.2 for $\eta=0.1$ and different values of α (the notation on the curves is the same as in Fig.1), and the point of application of the force is $\varphi_1=\pi$. It is seen that as α increases a decrease occurs in the amplitude of the velocity in the region where the mass is clamped. Therefore, relations (3.1) and (3.2) can be used for the vibrational diagnostics of inhomogeneous CS, they show the nature of the changes in both the eigenfrequencies and the distribution of the vibrational velocity as a function of the inhomogeneity parameters.

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METHOD OF EXTRACTING SINGULARITIES IN THE PROBLEM OF THE HYDROELASTIC VIBRATIONS OF A SHELL EXCITED BY CONCENTRATED FORCES*

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An asymptotic justification for the procedure /1, 2/ of matching the integrals of the vibrations of a shell and the Helmholtz equations for the acoustic pressure is given in the example of the problem of the vibrations of a closed spherical shell in an infinite medium, excited by forces applied at the poles of the shell. The order of constructing the approximate solution, based on replacement of the fluid influence by several apparent masses each of which is related to a specific integral of the shell equations, and on extraction of the singularities of the solution at the point of application of the force, is traced. The results are compared with the exact solution of the problem in the form of series in spherical functions /3/.

1. We will write the original system of equations of the axisymmetric vibrations of a spherical shell and an ideal compressible fluid while separating out the time dependence, given in the form $e^{-i\omega t}$ in the functions of the load Z, the acoustic pressure p and the shell displacements u and w

$$(1+\nu)\, w_{,\theta} - \Phi_{,\theta} - (1-\nu)\,\mu_0 u = 0, \quad \Phi = \sin^{-1}\theta\,(u\sin\theta)_\theta, \tag{1.1}$$

$$[\nabla^4 - (1-\nu)\nabla^2]\, w - c_*^2\left[\frac{\Phi}{1-\nu} - \left(\frac{2}{1-\nu} - \alpha_0^2\right) w\right] = \frac{r_0^4}{D}(Z - p|_S)$$

$$\nabla^2 p + (r^2 p_{,r})_{,r} + (kr)^2 p = 0, \quad \lim_{r\to\infty} r\,(p_{,r} - ikp) = 0$$

$$p_{,r}|_S = \omega^2 \rho w, \quad \alpha_0 = \omega r_0 (\rho_0/E)^{1/2}, \quad k = \omega/c$$

$$\mu_0 = 1 + \alpha_0^2 (1+\nu), \quad D = 2Eh^3/[3(1-\nu^2)]$$

$$c_*^2 = 2Ehr_0^2/D, \quad \nabla^2 = (\ )_{,\theta\theta} + \operatorname{ctg}\theta\,(\ )_{,\theta}, \quad (\ )_{,x} = \partial/\partial x$$

Here r and θ are spherical coordinates ($r = r_0$ is the equation of the shell surface S), h is half the shell thickness, ω is the angular frequency of the vibrations, ρ_0, E, ν are the density, Young's modulus, and Poisson's ratio of the shell material, and ρ and c are the density and velocity of sound in the fluid.

One of the effective approximate methods of solving two-dimensional problems of the type (1.1) is to reduce them to a one-dimensional problem on the shell surface by using an exponential representation of the fluid pressure integrals in the neighbourhood of the shell

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